# Edge Modes in Gauge Theories:
## An Algebraic Perspective.


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# Setting the stage

- Manifold → A globally hyperbolic Lorentzian manifold $(M, g)$ of dim $n$.
  → It has order 2 corners (i.e. locally it is homeomorphic to $[0,\infty)^2 \times \mathbb{R}^{n-2}$ )

  → It admits a compact Cauchy surface $\Sigma$ with non-trivial boundary $\partial\Sigma \neq \phi$.
  [For simplicity, we take $\Sigma$ connected and $\partial\Sigma$ connected]

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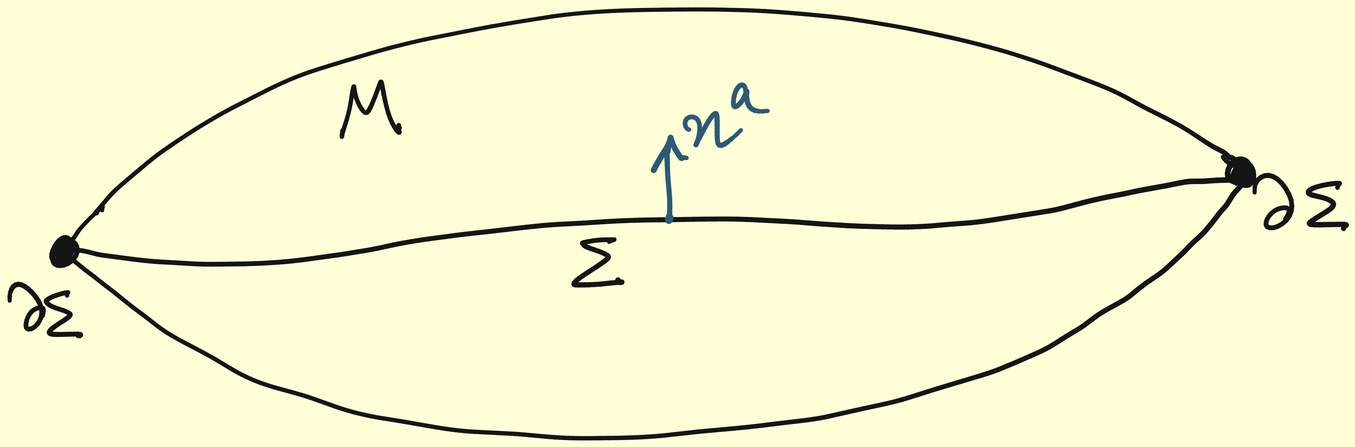
  → It admits a compact Cauchy surface $\Sigma$ with non-trivial boundary $\partial\Sigma \neq \phi$.
  [For simplicity, we take $\Sigma$ connected and $\partial\Sigma$ connected]

- **Field theory** → Some gauge theory on $(M, g)$.

  This talk :→ Electromagnetism, a little bit of linearized gravity.

  (If you invite me next year :)) :→ Linearized gravity, Yang-Mills theory, charged matter fields.

# Setting the stage



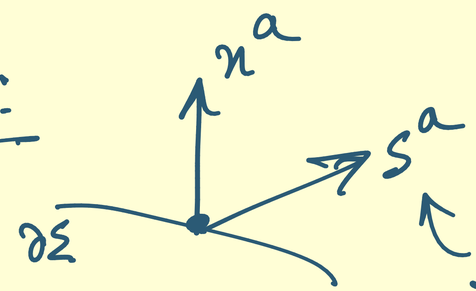

$n^a n_a = -1$

- $a, b, c, \ldots \rightarrow$ Spacetime indices

  $i, j, k, \ldots \rightarrow$ Indices on $\Sigma$.

- Wald's abstract index convention

On $\partial\Sigma$:




- $q_{ij}$ is induced metric on $\partial\Sigma$.

- $h_{ij}$ on $\Sigma$ and $D_i$ is Levi-Civita connection on $\Sigma$.

# Edge Modes: Why ?

- Gauge transformations which act trivially on the boundary are redundancies of the theory. ⟶ (1)
- But gauge transformations with non-trivial action on the boundary are not redundancies of the theory. ⟶ (2)

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Covariant phase space picture ⟵ (1) ⟶ Wilson line picture

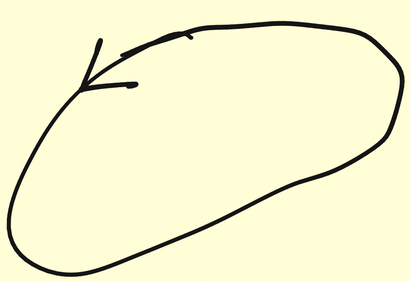
If $z$ is arbit. sol$^n$ and $g_\lambda$ is pure gauge:

$$W^\Sigma(\delta z, g_\lambda) = \delta Q_\lambda(z)$$

where $Q_\lambda$ is charge at $\partial\Sigma$ associated with $g_\lambda$.

If $Q_\lambda \equiv 0$, then $g_\lambda \in \mathrm{Rad}(W)$.

Redundancy!




Wilson loops

$$W[\gamma] = \mathrm{Tr}\left[\mathrm{P}\exp\left(-\oint_\gamma A_\mu dx^\mu\right)\right]$$

$$A_\mu \longmapsto A_\mu(\lambda)$$

$$W[\gamma] \longmapsto W[\gamma]$$

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- Gauge transformations which act trivially on the boundary are redundancies of the theory. ⟶ (1)
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Covariant phase space picture ⟵ (2) ⟶ Wilson line picture

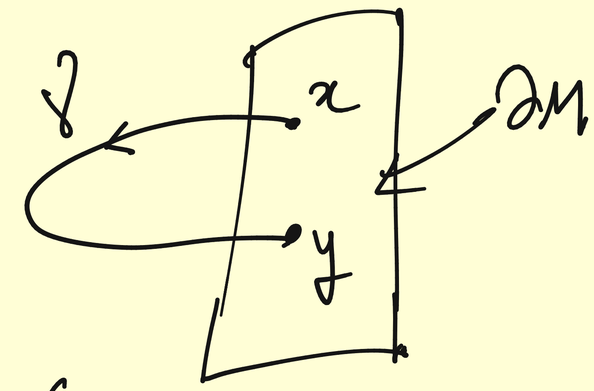
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$$W^\Sigma(\delta z, g_\lambda) = \delta Q_\lambda(z)$$

where $Q_\lambda$ is charge at $\partial\Sigma$ associated with $g_\lambda$.

If $Q_\lambda \neq 0$; then **not** a redundancy!

LARGE GAUGE TRANSFORMATIONS.




Open Wilson lines

$$W[\gamma] = \mathrm{Pexp}\left(-\int_\gamma A_\mu dx^\mu\right)$$

$$A_\mu \longmapsto A_\mu(\lambda)$$

$$W[\gamma] \longmapsto \lambda(x)\, W[\gamma]\, \lambda(y)^{-1}$$

# Edge Modes: Why ?

- Gauge transformations which act trivially on the boundary are redundancies of the theory. ⟶ (1)
- But gauge transformations with non-trivial action on the boundary are not redundancies of the theory. ⟶ (2)

Qn:- Can we make large gauge transformations redundancies of our theory ?

Ans:- Extend our theory to include new "surface degrees of freedom".

↓

Make these surface degrees of freedom "talk to the gauge field"

↓

Resultant extended theory (gauge theory + surface fields) has extended large gauge transf. as redundancies. "Edge modes".

And "why should we"?
(I will answer this implicitly, else ask me at the end)

# Electromagnetism: Classical picture :-

Let's do source-less for simplicity. [If sourced, simply take a background soln, subtract the background, and then proceed as follows]

- Initial data on $\Sigma$:

$$(a_i, e^i)$$

Constraint → (Linearized) Gauss constraint: $D_i e^i = 0$

# Electromagnetism: Classical picture :-

Let's do source-less for simplicity.

- Initial data on $\Sigma$:

$(a_i, e^i)$

Constraint → (Linearized) Gauss constraint: $D_i e^i = 0$

$\mathbb{V}_1 := C^\infty(T^*\Sigma \oplus T\Sigma)$ ⇝ $L^2$-norm on it → $G$

$z^{(1)} \equiv (a_i^{(1)}, e^{(1)i})$ ; $z^{(2)} \equiv (a_i^{(2)}, e^{(2)i})$

$$G(z^{(1)}, z^{(2)}) = \int_\Sigma d^3x \sqrt{h} \left(a_i^{(1)} a^{(2)i} + e_i^{(1)} e^{(2)i}\right)$$

$\mathbb{V}_2 := C^\infty(\Sigma)$ → $L^2$-norm $\langle \cdot, \cdot \rangle$

$$\langle f, g \rangle = \int_\Sigma d^3x \sqrt{h}\, f g$$

[N.B.: Neither $(\mathbb{V}_1, G)$, nor $(\mathbb{V}_2, \langle \cdot, \cdot \rangle)$ is complete]

# Electromagnetism: Classical picture :-

Let's do source-less for simplicity.

- Initial data on $\Sigma$:

$(a_i, e^i)$

Constraint → (Linearized) Gauss constraint: $D_i e^i = 0$

$\mathbb{V}_1 := C^\infty(T^*\Sigma \oplus T\Sigma)$ ⇝ Symplectic bilinear form on it → $W_{EM}$

$\mathbb{V}_2 := C^\infty(\Sigma)$

$z^{(1)} \equiv (a_i^{(1)}, e^{(1)i})$ ; $z^{(2)} \equiv (a_i^{(2)}, e^{(2)i})$

$$W_{EM}(z^{(1)}, z^{(2)}) = \int_\Sigma d^3x \sqrt{h}\left(a_i^{(1)} e^{(2)i} - a_i^{(2)} e^{(1)i}\right)$$

[ $U : (a_i, e^i) \mapsto (-e_i, a^i)$ can relate:

$W_{EM}(z^{(1)}, U z^{(2)}) = G(z^{(1)}, z^{(2)})$ ]

# Electromagnetism: Classical picture :-

Let's do source-less for simplicity.

- Initial data on $\Sigma$:

$(a_i, e^i)$

Constraint → (Linearized) Gauss constraint: $D_i e^i = 0$

$\mathbb{V}_1 := C^\infty(T^*\Sigma \oplus T\Sigma)$

$\mathbb{V}_2 := C^\infty(\Sigma)$

$L : \mathbb{V}_1 \longrightarrow \mathbb{V}_2$

$(a_i, e^i) \longmapsto D_i e^i$

So, constraint-satisfying data: $\ker L$.

# Electromagnetism: Classical picture :-

Let's do source-less for simplicity.

- Initial data on $\Sigma$:

$(a_i, e^i)$

Constraint $\longrightarrow$ (Linearized) Gauss constraint: $D_i e^i = 0$

$\mathbb{V}_1 := C^\infty(T^*\Sigma \oplus T\Sigma)$

$\mathbb{V}_2 := C^\infty(\Sigma)$

$\Gamma : \mathbb{V}_2 \longrightarrow \mathbb{V}_1$

$\Lambda \longmapsto (D_i \Lambda, 0)$

"Gauge map"

Pure gauge

$[\Gamma = UL^*$ actually$]$

# Electromagnetism: Classical picture :-

Let's do source-less for simplicity.

- Initial data on $\Sigma$:

$$(a_i, e^i)$$

Constraint $\longrightarrow$ (Linearized) Gauss constraint: $D_i e^i = 0$

Charge associated with gauge transf:- $\left[ z \in \ker C \iff D_i e^i = 0 \right]$

$$Q_\Lambda(z) = W_{EM}\left((D_i\Lambda, 0), (a_i, e^i)\right)$$

$$= \int_\Sigma d^3x \sqrt{h} (D_i\Lambda) e^i = \int_{\partial\Sigma} d^2x \sqrt{q}\, \Lambda (s_i e^i) - \int_\Sigma d^3x \sqrt{h}\, \Lambda \underbrace{D_i e^i}_{=0}$$

$$\boxed{Q_\Lambda(z) = \int_{\partial\Sigma} d^2x \sqrt{q}\, \Lambda (s_i e^i)}$$

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$$\boxed{Q_\Lambda(Z) = \int_{\partial\Sigma} d^2x \sqrt{q}\, \Lambda(S_i e^i)}$$

(Small) gauge transformations:- $\Lambda|_{\partial\Sigma} = c \in \mathbb{R}$ ; as then $Q_\Lambda(Z) = 0$

Large gauge transformation: $\Lambda|_{\partial\Sigma} \in C^\infty(\partial\Sigma)/\mathbb{R}$ .

# Electromagnetism: Classical picture :-

Let's do source-less for simplicity.

- Initial data on $\Sigma$:

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Constraint $\longrightarrow$ (Linearized) Gauss constraint: $D_i e^i = 0$

$\rightarrow \mathbb{V}_1 := C^\infty(T^*\Sigma \oplus T\Sigma)$

$\mathbb{V}_2 := C^\infty(\Sigma)$

$$\mathbb{V}_2^{(0)} = \{\Lambda \in \mathbb{V}_2 \mid \Lambda|_{\partial\Sigma} = c \in \mathbb{R}\}$$

(So, $\Gamma\mathbb{V}_2^{(0)}$ is the set of (small) gauge transformations).

# Electromagnetism: Classical picture :-

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Split-1 :-

All possible constraint-violating data.

$\mathbb{V}_1 \cong \mathcal{L}^* \mathbb{V}_2^{(0)} \oplus \ker \mathcal{L}$

Split-2:

$\ker \mathcal{L} \cong \Gamma \mathbb{V}_2^{(0)} \oplus (\ker \mathcal{L} \cap \ker \Gamma^*)$

ALL SPLITS ARE G-ORTHOGONAL.

# Electromagnetism: Classical picture :-

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$$\mathbb{V}_2 := C^\infty(\Sigma)$$

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**Split-1 :-**

All possible constraint-violating data.

$$\mathbb{V}_1 \cong \mathcal{L}^* \mathbb{V}_2^{(0)} \oplus \underbrace{\text{Ker}\,\mathcal{L}}$$

**Split-2 :**

$$\text{Ker}\,\mathcal{L} \cong \Gamma\, \mathbb{V}_2^{(0)} \oplus (\text{Ker}\,\mathcal{L} \cap \text{Ker}\,\Gamma^*)$$

ALL SPLITS ARE $G_1$-ORTHOGONAL.

$(\text{Ker}\,\mathcal{L}, W_{EM})$ forms a pre-symplectic space.

$\text{Rad}\left(W_{EM}|_{\text{Ker}\,\mathcal{L}}\right) = \Gamma\, \mathbb{V}_2^{(0)}$.

$\mathcal{P}_1 := \text{Ker}\,\mathcal{L} / \Gamma\, \mathbb{V}_2^{(0)} \cong (\text{Ker}\,\mathcal{L} \cap \text{Ker}\,\Gamma^*)$ is now (small) gauge-reduced phase space with $W_{EM}$ as symplectic form.

# Electromagnetism: Classical picture :-

$V_1 \cong \mathcal{L}^* V_2^{(0)} \oplus \Gamma V_2^{(0)} \oplus P_1$ ; $(P_1, W_{EM})$ is symplectic phase space.

Split-3:

$$(P_1, W_{EM}) \cong (P_1^{bulk}, W_{EM}^{bulk}) \oplus (P_1^{bdy}, W_{EM}^{bdy})$$

Bulk-boundary split of EM phase space.

$\downarrow$

$\widetilde{F} \oplus \widetilde{F}$

where $\widetilde{F} \cong C^\infty(\partial\Sigma)/\mathbb{R}$

"All smooth boundary functions modulo constants"

$$\cong \left\{ f \in C^\infty(\partial\Sigma) \,\middle|\, \int_{\partial\Sigma} d^2x \sqrt{q} f = 0 \right\}$$

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$$(P_1, W_{EM}) \cong (P_1^{bulk}, W_{EM}^{bulk}) \oplus (P_1^{bdy}, W_{EM}^{bdy})$$

$$\downarrow$$

$$\tilde{F} \oplus \tilde{F}$$

where $\tilde{F} \cong C^\infty(\partial\Sigma)/\mathbb{R}$

$$\cong \left\{ f \in C^\infty(\partial\Sigma) \,\middle|\, \int_{\partial\Sigma} d^2x \sqrt{q} f = 0 \right\}$$

Introduce

Surface fields :- $\omega \equiv (\varphi, \tau) \in \tilde{F} \oplus \tilde{F}$

with $$W_{surface}(\omega^{(1)}, \omega^{(2)}) = \int_{\partial\Sigma} d^2x \sqrt{h}\, \varphi^{(1)} \tau^{(2)} - \varphi^{(2)} \tau^{(1)}$$

# Electromagnetism: classical picture :-

$\mathbb{V}_1 \cong \mathcal{L}^* \mathbb{V}_2^{(0)} \oplus \Gamma \mathbb{V}_2^{(0)} \oplus \mathcal{P}_1$ ; $(\mathcal{P}_1, W_{EM})$ is symplectic phase space.

$$(\mathcal{P}_1, W_{EM}) \cong (\mathcal{P}_1^{bulk}, W_{EM}^{bulk}) \oplus (\mathcal{P}_1^{bdy}, W_{EM}^{bdy})$$

Introduce

Surface fields :- $\omega \equiv (\varphi, \tau) \in \mathcal{F} \oplus \mathcal{F} \equiv \widetilde{\mathbb{V}}$

with $W_{surface}(\omega^{(1)}, \omega^{(2)}) = \int_{\partial\Sigma} d^2x \sqrt{h}\, \varphi^{(1)} \tau^{(2)} - \varphi^{(2)} \tau^{(1)}$

- Extend the theory $\left(\widetilde{\mathcal{P}} = \mathcal{P}_1 \oplus \widetilde{\mathbb{V}},\ \widetilde{W} = W_{EM} \oplus W_{surf}\right)$ $\rightarrow g_\lambda$
- Extend the large gauge transf. for EM + surf. theory $\longrightarrow$ $\widetilde{g}_\lambda \equiv (\underbrace{D_i(H_D\lambda)}_{a_i}, \underbrace{0}_{e^i}, \underbrace{\lambda}_{\varphi}, \underbrace{0}_{\tau})$

where $\lambda \in \mathcal{F}$

$$\widetilde{\mathcal{G}} := \{\widetilde{g}_\lambda \mid \lambda \in \mathcal{F}\}$$

# Electromagnetism: Classical picture :-

- $\mathcal{V}_1 \cong \mathcal{L}^* \mathcal{V}_2^{(0)} \oplus \Gamma \mathcal{V}_2^{(0)} \oplus \mathcal{P}_1$ ; $(\mathcal{P}_1, \omega_{EM})$ is symplectic phase space.

- $(\mathcal{P}_1, \omega_{EM}) \cong (\mathcal{P}_1^{bulk}, \omega_{EM}^{bulk}) \oplus (\mathcal{P}_1^{bdy}, \omega_{EM}^{bdy})$

- Surface fields :- $\widetilde{\mathcal{V}} \equiv \mathcal{F} \oplus \bar{\mathcal{F}}$ with $\omega_{surf.}$ ; $\widetilde{\mathcal{G}}$ → subspace of extended large gauge.

Pre-symplectic space of extended theory : $\mathcal{P}_{ext} := \widetilde{\mathcal{G}}^{\perp_{\widetilde{\omega}}} = \left\{ (a_i, e^i, \varphi, \tau) \,\middle|\, (a_i, e^i) \in \mathcal{P}_1 , \; \tau = -s_i e^i|_{\partial\Sigma} \right\}$

↑ It turns out to be

↑ Surface fields "talk to gauge fields" (Part-I)

# Electromagnetism: classical picture :-

- $\mathcal{V}_1 \cong \mathcal{L}^* \mathcal{V}_2^{(0)} \oplus \Gamma \mathcal{V}_2^{(0)} \oplus \mathcal{P}_1$ ; $(\mathcal{P}_1, \omega_{EM})$ is symplectic phase space.

- $(\mathcal{P}_1, \omega_{EM}) \cong (\mathcal{P}_1^{bulk}, \omega_{EM}^{bulk}) \oplus (\mathcal{P}_1^{bdy}, \omega_{EM}^{bdy})$

- Surface fields :- $\tilde{\mathcal{V}} \equiv \mathcal{T} \oplus \mathcal{F}$ with $\omega_{surf.}$ ; $\tilde{\mathcal{G}}$ → subspace of extended large gauge.

Pre-symplectic space of extended theory : $\mathcal{P}_{ext} := \tilde{\mathcal{G}}^{\perp_{\tilde{\omega}}} = \left\{ (a_i, e^i, \varphi, \tau) \,\middle|\, (a_i, e^i) \in \mathcal{P}_1 , \; \tau = -s_i e^i \big|_{\partial\Sigma} \right\}$

⊙ One can show that $Rad(\tilde{\omega}) = \tilde{\mathcal{G}}$ ← Joint (i.e. extended) large gauge are redundancies !

So, $\left( \mathcal{P}_{ext} / \tilde{\mathcal{G}} , \tilde{\omega} \right)$ is symplectic.

# Electromagnetism: Classical picture :-

- $\mathcal{V}_1 \cong \mathcal{L}^* \mathcal{V}_2^{(0)} \oplus \Gamma \mathcal{V}_2^{(0)} \oplus \mathcal{P}_1$ ; $(\mathcal{P}_1, \omega_{EM})$ is symplectic phase space.

- $(\mathcal{P}_1, \omega_{EM}) \cong (\mathcal{P}_1^{bulk}, \omega_{EM}^{bulk}) \oplus (\mathcal{P}_1^{bdy}, \omega_{EM}^{bdy})$

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- Pre-symplectic space of extended theory : $\mathcal{P}_{ext} := \widetilde{\mathcal{G}}^{\perp_{\widetilde{\omega}}} = \left\{ (a_i, e^i, \varphi, \tau) \,\middle|\, (a_i, e^i) \in \mathcal{P}_1, \; \tau = -s_i e^i|_{\partial\Sigma} \right\}$

- $(\mathcal{P}_{ext}/\widetilde{\mathcal{G}}, \widetilde{\omega})$ is symplectic.

- $\mathcal{P}_{ext}/\widetilde{\mathcal{G}} \overset{\text{can be shown}}{\cong} \left\{ (a_i, e^i, \varphi, \tau) \,\middle|\, (a_i, e^i) \in \mathcal{P}_1, \; \tau = -s_i e^i|_{\partial\Sigma}, \; \varphi = -s_i a^i|_{\partial\Sigma} \right\}$

"Surface fields" talk to "gauge fields" (Part-II)

# Electromagnetism: Classical picture :-

## TL; DR :-

- Small gauge reduction gives symplectic phase space $(P_I, W_{EM})$ which retain large gauge transformations as symmetries, but not redundancies.
- Surface fields $(\varphi, \tau)$ are put at the boundary and large gauge transformations are extended to them.
- Final large gauge reduced phase space $\left(P_{ext}/\tilde{G}, \tilde{W}\right)$ has surface fields fully constrained by EM fields:

$$\varphi = -s^i a_i|_{\partial\Sigma}, \quad \tau = -s_i e^i|_{\partial\Sigma}.$$

Also importantly: $\left(P_{ext}/\tilde{G}, \tilde{W}\right) \underset{\text{symplectomorphic}}{\cong} \left(P_I, W_{EM}\right)$

# Electromagnetism: Quantum picture :-

- $\left(\mathcal{P}_{ext}/\tilde{G}\,,\ \tilde{W}\right) \cong \left(\mathcal{P}_1, W_{EM}\right) \cong \left(\mathcal{P}_1^{bulk}, W_{EM}^{bulk}\right) \oplus \left(\mathcal{P}_1^{bdy}, W_{EM}^{bdy}\right)$

Now $\mathcal{A}_{EM} := Weyl\left(\mathcal{P}_1, W_{EM}\right) \cong Weyl\left(\mathcal{P}_1^{bulk}, W_{EM}^{bulk}\right) \otimes Weyl\left(\mathcal{P}_1^{bdy}, W_{EM}^{bdy}\right)$

$\quad\quad\quad

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$\quad\quad\quad

# Electromagnetism: Quantum picture :-

- $\mathcal{A}_{EM} \cong \mathcal{A}_{EM}^{bulk} \otimes \mathcal{A}_{EM}^{bdy}$
- $\mathcal{A}_{ext.} = \mathcal{A}_{EM} \otimes \mathcal{A}_{surface}$
- $\mathcal{A}_{inv} = (\mathcal{A}_{ext.})^{\tilde{\alpha}(\lambda)}$ ; $\tilde{\alpha}(\lambda) \rightarrow$ Adjoint action of unitaries implementing joint large gauge transformations.

Relativisation map: $\yen \rightsquigarrow$ expresses EM observables w.r.t. surface fields

$$\yen : \mathcal{A}_{EM} \longrightarrow \mathcal{A}_{inv}$$

$$\yen(\mathcal{A}_{EM}) \cong \mathcal{A}_{EM}$$

# Linearized Gravity : Classical treatment

| | EM | Linearized gravity |
|---|---|---|
| $V_1$ | $C^\infty(T^*\Sigma \oplus T\Sigma)$ | $C^\infty(S^2T^*\Sigma \oplus S^2T\Sigma)$ |
| $V_2$ | $C^\infty(\Sigma)$ | $C^\infty(\Sigma) \oplus C^\infty(T\Sigma)$ |
| $\mathcal{L}$ | (Linearized) Gauss constraint | Linearized ADM constraints |
| $\Gamma$ | $f \longmapsto (D_i f, 0)$ | $(C, X^i) \longmapsto (\gamma_{ij}, \beta^{ij})_{\text{gauge}}$ <br> $\gamma_{ij} = 2D_{(i}X_{j)} + 2K_{ij}C$ |

# Linearized Gravity : Classical treatment

| | EM | Linearized gravity |
|---|---|---|
| $V_1$ | $C^\infty(T^*\Sigma \oplus T\Sigma)$ | $C^\infty(S^2T^*\Sigma \oplus S^2T\Sigma)$ |
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| $\mathcal{L}$ | (Linearized) Gauss constraint | Linearized ADM constraints |
| $\Gamma$ | $f \longmapsto (D_i f, 0)$ | $(C, X^i) \longmapsto (\gamma_{ij}, \beta^{ij})_{\text{gauge}}$ <br> $\gamma_{ij} = 2D_{(i}X_{j)} + 2K_{ij}C$ |

Problems :- Splits -1,2 worked because for EM, $\mathcal{L}\mathcal{L}^* = -D^2 = \Gamma^*\Gamma$ and we used existence and uniqueness of Poisson eq$^n$ sol$^n$s with Dirichlet/Neumann/Robin boundary cond$^n$s.

For gravity, $\mathcal{L}\mathcal{L}^*$ and $\Gamma^*\Gamma$ ⟶ not so simple!

## Looking Ahead :-

(1) Edge modes and their QRF interpretation for linearized gravity

(2) Taking charged matter fields, like QED.

(3) Interacting QFTs like Yang-Mills

Thank You !

