Inverse Scattering and the Backlund Transform

Sunday, 18 January 2026 15:30

Integrability in Classical Mechanics

Let M be phase space and (q_i, p_i), $i = 1, \dots, n$ be coordinates on this phase space.

Our dynamical variables are $f : M \times \mathbb{R} \to \mathbb{R}$ where $f = f(q, p, t)$.

We define a Lie bracket with a product rule called the Poisson bracket

$$\{f, g\} = \partial_{q_k} f \, \partial_{p_k} g - \partial_{p_k} f \, \partial_{q_k} g$$

f, g are said to be in involution if $\{f, g\} = 0$

Our coordinates obey, $\{q_i, q_j\} = \{p_i, p_j\} = 0$, $\{q_i, p_j\} = \delta_{ij}$.

Dynamics are defined by

$$\dot{f} = \partial_t f + \{f, H\}$$

where $H = H(q, p, t)$ is the Hamiltonian. We have $2n$ ODE's

$$\dot{p}_i = -\partial_{q_i} H, \quad \dot{q}_i = \partial_{p_i} H$$

which are uniquely determined by $(q(0), p(0))$.

The above Hamiltons equations lead to volume elements being conserved in phase space.

Definition 1.1.1)

A function f which has $\dot{f} = 0$ is a first integral if

A function f which has $\dot{f}=0$ is a first integral if